Rectangular fin: spacing optimization and new radiative flux evaluation method.

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Abstract: - Fin spacing is chosen to maximize heat transmission rate. Maximization is attained as a compromise between two opposite trends: fin surface maximization and adduction coefficient maximization. In this paper a new relation for the optimum spacing between rectangular fins is proposed. The relation was obtained keeping into account non-unitary fin efficiency. Demonstration is achieved by a theoretical approach which regards only rectangular fins. Results were verified also by numerical simulations. An original method to evaluate radiative heat flux is proposed too. The method allows to obtain an original simple radiative heat flux equation. Numerical simulations and theoretical evaluations show that the proposed convective flux optimization method and the model for radiative flux estimation are respectively suitable for high length and high temperature rectangular fins.

Key-Words: - heat flux, fin, spacing, efficiency, adduction.

1 Introduction

Electronic systems (power electronics, CPU cooling) performances and reliability depend on working temperature, thus cooling plays a fundamental role which is becoming more and more important because of device miniaturization. Fin arrays are one of the most common system for devices cooling. Fin spacing is chosen to maximize heat transmission rate. Maximization is attained as a compromise between two opposite trends:

- fin surface maximisation;
- adduction coefficient maximisation.

Fin surface increase by diminishing spacing; adduction coefficient increase by increasing spacing. Optimal fin spacing has been found assuming the following approximations:

a) convection coefficient is uniform along the fin [1];
b) radiative heat transfer rate is negligible;
c) fin efficiency is assumed unitary.

In this paper it is demonstrated that spacing may be furthermore optimized if fin efficiency is kept into account; thus only approximations a) and b) are still taken into account. Demonstration is achieved by a theoretical approach and by numerical simulations. Optimized spacing allows to decrease thermal resistance maintaining the same fin system dimension. This result may be important for electronic applications where high heat transfer rates are produced by very small devices.

Furthermore, a new method for fin radiative heat transfer evaluation is proposed. The method proposes a uniform temperature fin model. Model result depends on a calibrating parameter value. It is demonstrated that the calibrating parameter is constant for any fin configuration. Simulations validated theoretical findings.

2 Optimal Rectangular Fin Spacing

Heat transfer rate transferred from an hot surface (a fin system) to the surrounding environment is given by two contributions:

$$q = q_c + q_r \tag{1}$$

When temperature difference is small enough the radiation term q_r may be neglected [1]. In order to increase heat transfer rate fin surface and convection coefficient h may be both increased. A generic thin fin temperature distribution is ruled by the following relation if only convective heat transfer rate is considered:

$$\frac{T - T_\infty}{T_p - T_\infty} = \frac{cosh(m(L-x)) + \frac{h}{m\lambda} sinh(m(L-x))}{cosh(mL) + \frac{h}{m\lambda} sinh(mL)} \tag{2}$$

Thus, heat transfer rate is:

$$Q_d = \sqrt{hP\lambda A} \cdot (T_s - T_\infty) \cdot \frac{sinh(mL) + \frac{h}{m\lambda} cosh(mL)}{cosh(mL) + \frac{h}{m\lambda} sinh(mL)} \tag{3}$$

Fin efficiency is defined as it follows:

$$\eta = \frac{Q_d}{Q_{id}} = \frac{Q_d}{h \cdot S \cdot (T_p - T_\infty)} \tag{4}$$

On such a condition, fin heat flux rate may be found by the following relation:

$$Q_d = \eta \cdot h \cdot S \cdot (T_p - T_\infty) \qquad (5)$$

When a rectangular fin is characterized by *b* much greater than *t*, it may be written:

$$m = \sqrt{\frac{hP}{\lambda A}} \cong \sqrt{\frac{2h}{t\lambda}} \qquad (6)$$

Furthermore, fin efficiency is well approximated by the following relation [2]:

$$\eta = \frac{tgh(mL)}{mL} \qquad (7)$$

A rectangular fin system is constituted by an array of cavities as it is shown in Fig.1.

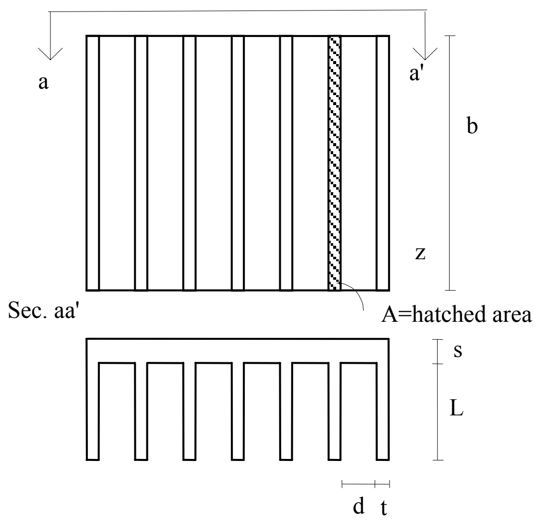

Fig.1: Rectangular fin system scheme.

On natural convection, convective coefficient depends on the dimension *d* of each cavity according to the following laws [3, 4]:

$$h = \frac{Nu \cdot \lambda_a}{d} \qquad (8)$$

where

$$Nu = \frac{1}{\sqrt{\frac{576}{(Ra')^2} + \frac{2.873}{(Ra')^{1/2}}}}, \; Ra' = Ra \cdot \frac{d}{b} \qquad (9)$$

Eq. (8) states that convection coefficient *h* increase when *d* increase. Anyway, global heat transfer rate increases by increasing total fin surface which means to increase the number of cavity corresponding to a reduction of *d*.

In order to maximize convective heat transfer it is needed to get an optimal fin spacing. Let introduce the following statements:

$$y = \frac{d}{b}$$
$$R = \frac{g \cdot \beta \cdot \Delta T \cdot b^3}{\nu^2} Pr \qquad (10)$$

Eq. (10) yields that:

$$Ra' = R \cdot y^4 \qquad (11)$$

For obtaining optimal spacing it is assumed in literature [5] that the fin efficiency is 1; such an assumption allows to write fin convective heat transfer rate as follows:

$$q_c = \frac{2 \cdot (z + d) \cdot L \cdot \lambda_a \cdot (T_p - T_\infty)}{b\sqrt{\frac{576}{R^2 \cdot y^4} + y^2 \frac{2.873}{\sqrt{R}}}} \qquad (12)$$

It may be shown that maximum value of q_c is attained when [6]:

$$y = 2.71 \cdot R^{-1/4} \rightarrow d_{ott} = 2.71 \cdot b \cdot R^{-1/4} \qquad (13)$$

3 Further Convective Heat Transfer Optimization

Relation (13) was found by considering a unitary fin efficiency. If efficiency is introduced a further optimization of fin spacing may be attained. Fin efficiency can be conveniently approximated by the following relation:

$$\eta = \frac{1}{1 + \frac{1}{3}(mL)^2} \qquad (14)$$

It may be shown that Eq. (14) approximates fin efficiency better than Taylor series truncated to the third term. Error committed using Eq. (14) on behalf of Eq. (7) is below 10% when *mL* is below 1.5 (see appendix A).
It may be simply shown that convective fin heat rate is:

$$q_c = 2 \cdot \left(\frac{z}{d} + 1\right) \cdot (L \cdot b) \cdot h \cdot \eta \cdot (T_p - T_\infty) \qquad (15)$$

Eq. (15) may be rewritten introducing equations (14) and (8) as follows:

$$q_c = \frac{2\cdot(z+d)\cdot L\cdot\lambda_a\cdot(T_p - T_\infty)}{b\cdot\sqrt{\frac{576}{R^2 y^4} + y^2\frac{2.873}{\sqrt{R}} + \frac{2\lambda_a L^2}{3\lambda_w t}y}} \tag{16}$$

Maximizing q_c means to solve the following equation in terms of y:

$$\frac{d}{dy}\left(b\cdot\sqrt{\frac{576}{R^2 y^4} + y^2\frac{2.873}{\sqrt{R}}} + \frac{2\lambda_a L^2}{3\lambda_w t}\right) = 0 \tag{17}$$

It yields to the following parabolic equation on variable $u = y^6$:

$$\left(\frac{16\cdot 2.873\cdot\lambda_a^2 L^4}{9\cdot t^2\sqrt{R}\cdot\lambda^2} - \frac{4\cdot b^2\cdot 2.873^2}{R}\right)\cdot u^2 +$$
$$\left(\frac{16\cdot 576\cdot\lambda_a^2 L^4}{9\cdot t^2\cdot R^2\cdot\lambda^2} + \frac{16\cdot b^2\cdot 2.873\cdot 576}{R^{5/2}}\right)u - \tag{18}$$
$$\left(\frac{16\cdot b^2\cdot 576^2}{R^4}\right)$$

Thus, optimal spacing is given by:

$$d_{ott} = b\cdot\sqrt[6]{u_{ott}} \tag{19}$$

4 Calculation example

Heat flux rate of two rectangular CPU dissipater were compared; dissipaters fin spacing was determined by two different methods:

A) unitary fin efficiency method;
B) the proposed "true fin efficiency" method (Eq. (14)).

The following geometrical and thermal parameter were considered:

ΔT = 80 K;
λ_w = 100 [W m^{-1} K];
λ_a = 0,0261 [W m^{-1} K];
v^2 = 2,531x10^{-10} [m^4 s^{-2}];
$g\beta$ = 0,027 [m s^{-2} K^{-1}];
Pr = 0,701 (biatomic gas);
t = 0,001 m;
L = 0,14 m;
b = 0,08 m;
z = 0,0937 m.

In Fig.2 convective flux versus fin spacing is shown. The radiative heat transfer rate is neglected with respect to convective heat transfer rate given by Eq. (15) for the investigated dissipater working temperatures. Therefore, fin heat transfer rate may be assumed equal to the convective one.

Fin spacing obtained by A) method is:

$$d_A = 5{,}18\ mm \tag{20}$$

By Eq. (19), fin optimal spacing by B) method is:

$$d_B = 4{,}43\ mm \tag{21}$$

The optimal spacing d_B produces a 17 cavities dissipater; on the contrary, fin spacing attained by A) unitary efficiency method yields 15 cavities.

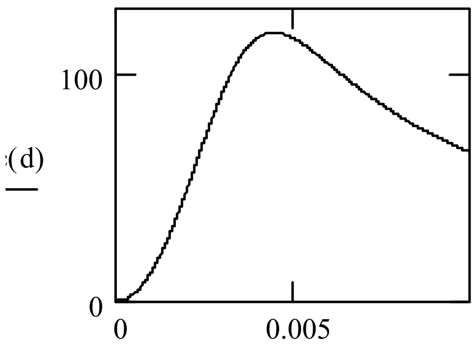

Fig.2: convective heat flux rate (W) given by Eq. (15) vs. fin spacing (m) for the investigated dissipater.

Heat flux increase due to B) method fin spacing optimization can be calculated by the following ratio:

$$\frac{q_B}{q_A} = \frac{2\cdot\left(\frac{z}{d_B}+1\right)\cdot(L\cdot b)\cdot(h\cdot\eta)_B}{2\cdot\left(\frac{z}{d_A}+1\right)\cdot(L\cdot b)\cdot(h\cdot\eta)_A} \tag{22}$$

Using fin spacing values in Eq. (20) and (21), ratio (22) results:

$$\frac{q_B}{q_A} = \frac{(z+d_B)\cdot d_A\cdot(h\cdot\eta)_B}{(z+d_A)\cdot d_B\cdot(h\cdot\eta)_A} = 1{,}03 \tag{23}$$

The proposed fin spacing leads to an approximately 3% heat flux increase. Heat flux decreases by using a fin spacing smaller and greater than the one given by Eq. (21).

A numerical simulation was conducted in order to validate the theoretical results. Finite volume models relative to A), B) fins were created [7-10]; each model were meshed into approximately 500000 tetrahedral elements. Mesh spacing was chosen equal to 0.5 mm near the fin and at the fin interior. Mesh spacing was increased in the air zone far from the fin one. The following boundary conditions were used:

- fin bottom wall temperature is 348 K.
- A pressure outlet condition was imposed for the air volume walls.

Simulation results showed an approximately 3.5% convective flux increase when the proposed B) method is used with respect to A) method. Heat fluxes values (see Table 1) are very close to the ones obtained by theoretical calculations. Also heat flux increase is very close to the one obtained by Eq. (22). Therefore, simulation results validate to have obtained better performances by B) method with respect to A) one. Simulation results confirm that d_B represents the optimal fin spacing for the investigated dissipater.

Table 1: comparison between convective heat fluxes obtained by theoretical calculations and simulations for the investigated rectangular fins.

Method	***Global Heat Fluxes (W)***	
	Theoretical Evaluation	*Simulation*
A) unitary fin efficiency	114.5	115.3
B) true fin efficiency	118.8	119.4

5 Original radiative flux evaluation method

Radiative heat transfer rate transmitted by a rectangular fin system is negligible for common working temperatures. Radiative heat flux may be relevant only for high working temperature and, as further shown, for particular geometrical configurations.

It is here proposed a general method for evaluating fins radiative heat flux when non-uniform walls temperature is considered.

The method may be used for fin systems when temperature is as high as radiative flux becomes non-negligible and for any other applications where rectangular cavities with non-uniform temperature walls are involved.

A rectangular fin is constituted by an array of cavities which emit heat flux to the external bodies. Global radiative flux transmitted by a rectangular fin system is the sum of single cavity radiative fluxes [11]. Single cavity radiative flux is found by adopting the following hypotheses:

a) cavity walls are perfectly diffusive gray bodies;
b) external bodies are modeled like a wall which completely closes each cavity;
c) external bodies surface temperature T_{ex} is uniform;
d) external bodies absorption coefficient is unitary;
e) wall radiances are uniforms.

Some different cases were investigated:

1. single wall temperature cavity (see Fig.3-a);
2. uniform wall temperature cavity, lateral walls temperature equal to the average temperature between air and fin bottom wall temperature (see Fig.3-b);
3. variable wall temperature cavity (see Fig.3-c);
4. uniform wall temperature model for variable wall temperature cavity (see Fig.3-d).

5.1 Case 1: Single wall temperature cavity

Hypotheses a) and e) allow to determine the cavity walls and external bodies radiances by solving, with respect to G_1 and G_2, the following two variables-two equations system:

$$\begin{cases} A_1 \cdot G_1 = A_1 \cdot J_1 + r_1 \cdot \left(A_2 \cdot G_2 \cdot F_{21} + A_1 \cdot (1-F_{12}) \cdot G_1 \right) \\ A_2 \cdot G_2 = A_2 \cdot J_2 + r_2 \cdot \left(A_1 \cdot G_1 \cdot F_{12} + A_2 \cdot (1-F_{21}) \cdot G_2 \right) \end{cases} \qquad (24)$$

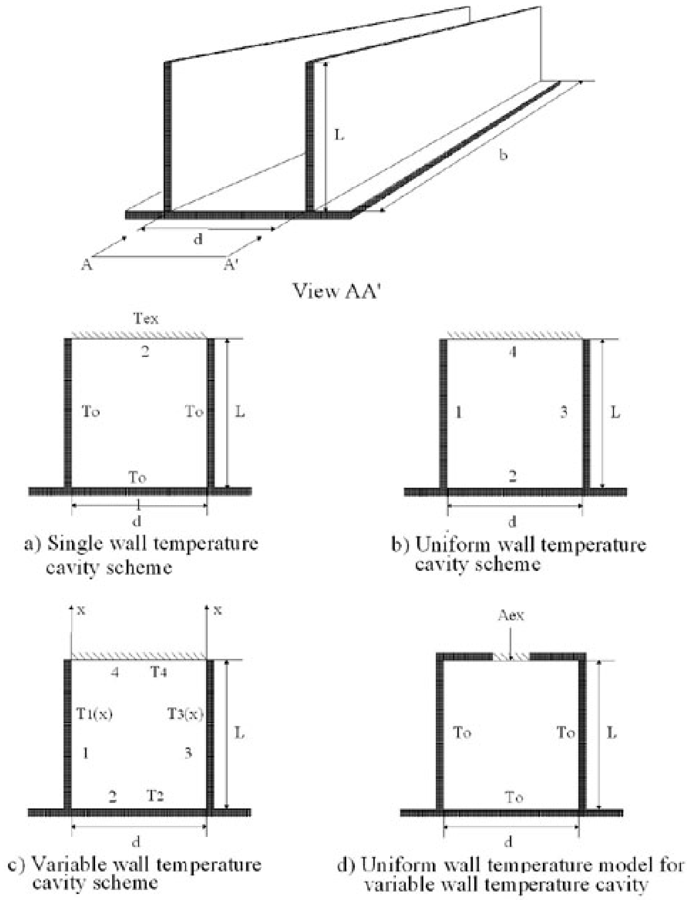

Fig.3: Single cavity temperature schemes.

The system may be solved by evaluating the following matrix:

$$\begin{pmatrix} G_1 \\ G_2 \end{pmatrix} = \begin{pmatrix} A_1(1-r_1(1-F_{12})) & -r_1 A_2 F_{21} \\ -r_2 A_1 F_{12} & A_2(1-r_2(1-F_{21})) \end{pmatrix}^{-1} \cdot \begin{pmatrix} A_1 J_1 \\ A_2 J_2 \end{pmatrix} \qquad (25)$$

Cavity wall temperature is assumed higher than external bodies one. Thus, heat flux transmitted by a single cavity to external bodies is given by:

$$q_{cav} = A_1 G_1 F_{12} - A_2 G_2 F_{21} \tag{26}$$

Equations (25) and (26) are calculated by observing that:

$$F_{21} \cong 1 \rightarrow F_{12} = \frac{A_2}{A_1} \rightarrow F_{11} = 1 - \frac{A_2}{A_1} \tag{27}$$

Eq. (26) may be written as follows by means of algebraic simplifications:

$$q_{cav} = A_2 \sigma_0 \cdot \frac{(T_0^4 - T_{ex}^4)}{\frac{1}{a_2} + \frac{A_2}{A_1}\left(\frac{1}{a_1} - 1\right)} \tag{28}$$

Cavity wall and external bodies areas are given by the following relations (see Fig.3-a):

$$A_1 = b(2L + d)$$

$$A_2 = bd + 2dL \tag{29}$$

Thus, Eq. (27) may be rewritten as follows:

$$q_{cav} = \sigma_0 \cdot \frac{(T_0^4 - T_{ex}^4) \cdot (bd + 2dL)}{1 + \frac{bd + 2dL}{b(2L + d)}\left(\frac{1}{a_1} - 1\right)} \tag{30}$$

The number n of cavities may be estimated as:

$$n \cong \frac{z}{d} + 1 \cong \frac{z}{d} \tag{31}$$

Thus, total fin system radiative flux q_r is given by the following equation:

$$q_r = q_{cav} \cdot n = \sigma_0 \cdot \frac{(T_0^4 - T_{ex}^4) \cdot z \cdot (b + 2L)}{1 + \frac{bd + 2dL}{b(2L + d)}\left(\frac{1}{a_1} - 1\right)} \cdot \tag{32}$$

Eq. (30) is as monotonic descending function of variable d into [0, z] range; it may be simply shown that maximum radiative heat flux is transmitted when d = 0. This absurd result is caused by the assumption that cavity wall temperature is considered uniform.

5.2 Case 2: uniform wall temperature cavity

In order to evaluate fin radiative heat flux, the walls temperature distribution may be considered uniform but not the same for each fin wall.

Referring to Fig.3-b, temperature distribution of 1 and 3 walls was calculated as the average of the air temperature T_4 and the bottom wall temperature T_2.
Thus,

$$T_1 = T_3 = \frac{T_2 + T_4}{2} \tag{33}$$

Each view factor must be calculated [12, 13]; the problem may be exactly solved. Radiative heat flux transmitted by the single cavity to the external body is given by:

$$q_{cav} = A_1 G_1 F_{14} + A_2 G_2 F_{24} + A_3 G_3 F_{34} - A_4 G_4 (F_{41} + F_{42} + F_{43}) \tag{34}$$

Radiances G_1, G_2, G_3 and G_4 may be calculated by means of the following relation:

$$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix} = \begin{bmatrix} A_1 & -r_1 A_2 F_{21} & -r_1 A_3 F_{31} & -r_1 A_4 F_{41} \\ -r_2 A_1 F_{12} & A_2 & -r_2 A_3 F_{32} & -r_2 A_4 F_{42} \\ -r_3 A_1 F_{13} & -r_3 A_2 F_{23} & A_3 & -r_3 A_4 F_{43} \\ -r_4 A_1 F_{14} & -r_4 A_2 F_{24} & -r_4 A_3 F_{34} & A_4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} \tag{35}$$

5.3 Case 3: Variable wall temperature cavity

A not uniform cavity wall temperature distribution is here proposed.

According to the reference system of Fig.3-c, cavity wall temperature distribution is assumed to be governed by the following equation:

$$T(x) = T_2 \cdot e^{-s \cdot x} \tag{36}$$

The s coefficient may be found by means of the following condition [14]:

$$s = \frac{1}{L} ln\left(\frac{T_2}{T_L}\right) \tag{37}$$

where T_L is obtained using Eq. (2) [1]:

$$T_L = \frac{(T_2 - T_4)}{cosh(mL) + \frac{h}{m\lambda} sinh(mL)} + T_4 \tag{38}$$

Assuming cavity walls to be a perfect diffusive gray body, radiative energy emitted by lateral wall 1 and 3 per unit area per unit solid angle is given by (see Fig.3-c):

$$J_1(\alpha,x)=J_{n,1}(x)\cdot cos(\alpha_1)=\frac{\sigma_0 a_1 T_1^4}{\pi}\cdot e^{-4\cdot s\cdot x}\cdot cos(\alpha_1)$$
$$J_3(\alpha,x)=J_{n,3}(x)\cdot cos(\alpha_3)=\frac{\sigma_0 a_1 T_3^4}{\pi}\cdot e^{-4\cdot s\cdot x}\cdot cos(\alpha_3) \quad (39)$$

Furthermore, radiative energy emitted by cavity base wall per unit area per unit solid angle is given by:

$$J_2(\alpha,x)=J_{n,2}(x)\cdot cos(\alpha_3)=\frac{\sigma_0 a_1 T_2^4}{\pi}\cdot cos(\alpha_3) \quad (40)$$

Equations (39) and (40) allow to introduce the following modified view factors which are referred to a not uniform temperature surfaces:

$$F_{12}=\frac{4sL}{A_1\pi\left(1-e^{-4sL}\right)}\cdot\int_{A_1}\int_{A_2}\frac{\cos(\alpha_1)\cdot\cos(\alpha_2)}{(R_{12})^2}\,dA_1\,dA_2$$
$$F_{13}=\frac{4sL}{A_1\pi\left(1-e^{-4sL}\right)}\cdot\int_{A_1}\int_{A_3}\frac{\cos(\alpha_1)\cdot\cos(\alpha_3)}{(R_{13})^2}\,dA_1\,dA_3 \quad (41)$$
$$F_{14}=\frac{4sL}{A_1\pi\left(1-e^{-4sL}\right)}\cdot\int_{A_1}\int_{A_4}\frac{\cos(\alpha_1)\cdot\cos(\alpha_4)}{(R_{14})^2}\,dA_1\,dA_4$$

with $F_{32}=F_{12}$; $F_{34}=F_{14}$; $F_{13}=F_{31}$.

The remaining six view factor are calculated by traditional method [15]; relations which involve view factors are the followings:

$$F_{24}=F_{42}$$
$$F_{41}=F_{43}=F_{21}=F_{23} \quad (42)$$

Radiative heat flux may be determined as well as for uniform temperature cavity; radiative heat flux transmitted by cavity to external bodies is given by Eq. (34). Radiances G_1, G_2, G_3 and G_4 may be calculated by means of the Eq. (35); J_1, J_2, J_3 and J_4 are obtained as follows:

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix}=\begin{bmatrix} \sigma_0 a_1 T_2^4\cdot\int_{A_1} e^{-4sx}dA_1 \\ A_2\sigma_0 a_2 T_2^4 \\ \sigma_0 a_3 T_2^4\cdot\int_{A_3} e^{-4sx}dA_3 \\ A_4\sigma_0 a_4 T_4^4 \end{bmatrix}=\begin{bmatrix} \sigma_0 a_1 T_2^4\cdot\frac{b}{4s}\left(1-e^{-4sL}\right) \\ A_2\sigma_0 a_2 T_2^4 \\ \sigma_0 a_3 T_2^4\cdot\frac{b}{4s}\left(1-e^{-4sL}\right) \\ A_4\sigma_0 a_4 T_4^4 \end{bmatrix} \quad (43)$$

Fin system radiative heat flux may be evaluated as the sum of each single cavity fluxes:

$$q_r=q_{cav}\cdot\frac{z}{d} \quad (44)$$

A comparison among the radiative heat fluxes obtained in cases 1-3 was carried out. A rectangular fin example is proposed where some geometrical and thermal parameters were varied to test each calculation method on different conditions.

Fin geometrical and thermal parameters are the following ones (Example a):

T_2 = 378 [K];
T_4 = 298 [K];
h = 10 [W m^{-2} K^{-1}];
λ = 200 [W m^{-1} K^{-1}];
z = 0.127 [m];
b = 0.075 [m];
L = 0.02 [m];
d = 0.00885 [m];
t = 0.0016 [m];
n = 12.

Four different conditions were obtained by changing respectively the following geometrical or thermal parameter:

Example b: d = 0.01094 m, n = 10;
Example c: T_2 = 348 K;
Example d: b = 0.12 m;
Example e: L = 0.03 m.

It may be observed in Table 2 that each calculation method (1, 2, 3) produces a quite different heat flux rate; this is due to the uniform wall temperature assumption for case 1 and 2 with respect to non-uniform wall temperature assumption for case 3.

Table 2: global radiative heat flux evaluation

	Calculation method			*Numerical simulation*
Total fin heat flux [W]	*Case 1*	*Case 2*	*Case 3*	
a	4.440	2.316	2.941	2.976
b	4.410	2.400	3.043	3.459
c	2.400	1.308	1.642	1.704
d	7.308	3.828	4.894	5.143
e	4.560	2.220	2.065	2.158

Thus a numerical simulation was carried out to compare calculation method. Fin was modeled by a 100000 tetrahedral elements mesh. Mesh spacing was chosen equal to 0.5 mm near the fin and at the fin interior. Mesh spacing was increased away from the fin. The following boundary conditions were used:

- fin bottom wall temperature equal to 378 K (348 K for Example c);

- a pressure outlet condition was imposed for the air volume walls.

Numerical results demonstrate that only non-uniform model gives viable radiative flux predictions which are very close to the numerical ones for each fin configuration.

5.4 Case 4: Uniform wall temperature model for variable wall temperature cavity

Difficult and huge calculations are required to determine radiative heat flux by non-uniform walls temperature Case 3 model. However it may be observed that a simplified formula may be introduced:

$$q_{cav} = \sigma_0 \cdot \frac{(T_0^4 - T_{ex}^4) \cdot A_2}{\frac{1}{a_2} + \frac{A_2}{A_1}\left(\frac{1}{a_1} - 1\right)} \tag{45}$$

where:

$$A_1 = b(2L + d) + (bd + 2dL)(1 - e^{-C_c \frac{L}{d}}) \tag{46}$$

and

$$A_2 = (bd + 2dL)e^{-C_c \frac{L}{d}} \tag{47}$$

In Eq. (47) C_c=0.34 is a calibrating not-dimensional parameter which has been determined by equalizing Eq. (45) and Eq. (34). Once C_c is known, it may be shown that Eq. (45) gives, for any fin configuration, a radiative flux which is very close to the one obtained by Case 3 method.

Furthermore, it may be observed that Eq. (45) represents a uniform temperature cavity model of the non-uniform temperature cavity. The entire fin system radiative flux rate is given by the following enclosed form relation:

$$q_r = \sigma_0 \cdot \frac{\left(T_0^4 - T_{ex}^4\right) \cdot z\left(b + 2L\right) \cdot e^{-0.34\frac{L}{d}}}{\frac{1}{a_2} + \frac{(bd + 2dL) \cdot e^{-0.34\frac{L}{d}}}{b(2L + d) + (bd + 2dL)(1 - e^{-0.34\frac{L}{d}})}\left(\frac{1}{a_1} - 1\right)} \tag{48}$$

When $b >> d$, Eq. (47) becomes:

$$A_2 = bd \cdot e^{-0.34 \cdot \frac{L}{d}} \tag{49}$$

Thus, Eq. (48) may be rewritten as follows:

$$q_r = \sigma_0 \cdot \frac{(T_0^4 - T_{ex}^4) \cdot z \cdot b \cdot e^{-0.34\gamma}}{1 + \frac{e^{-0.34\gamma}}{2\gamma + 2 - e^{-0.34\gamma}}\left(\frac{1}{a_1} - 1\right)} \tag{50}$$

where

$$\gamma = \frac{L}{d} \tag{51}$$

By Eq. (44) and Eq. (50) the fin radiative heat flux may be evaluated in a simplified way with respect to method 3 because a negligible error is introduced.

Radiative heat flux transmitted by the fin system described in paragraph 4 is negligible with respect to convective heat flux as may be demonstrated by Eq. (50). However radiative heat flux transmitted by a fin system must be taken into account for high working temperature or for peculiar fin geometrical configuration.

6 Conclusion

In this paper a new relation for rectangular fins array optimum spacing is proposed. The relation has been obtained keeping into account non-unitary fin efficiency.

The difference between heat dissipation rate calculated by literature method (unitary fin efficiency) and heat dissipation rate evaluated by the proposed method increases as fin height grows. Application on CPU dissipater shows heat flux increase up to 3%.

Radiative heat flux transmitted by a rectangular fin system is negligible for common working temperatures. However, radiative heat flux must be considered for high fin temperature and for particular geometrical configurations.

It is here proposed an original method to determine radiative heat flux when a non-uniform walls temperature fin array is considered which produces very accurate results.

Furthermore a simple relation is also proposed which can be used for fin arrays and for any other applications where cavity radiation phenomena are involved; the relation was validated by numerical simulations.

7 Appendix A: Fin Efficiency Approximation

In paragraph 1 a fin efficiency approximation is proposed to ease optimal fin spacing calculation. Considering the dimensions sketched in Fig.1, fin efficiency is expressed by the following relation:

$$\eta = \frac{tgh(mL)}{mL} \tag{52}$$

Using Eq. (52) Taylor series expansion, truncated to the third component, fin efficiency can be approximated as follows:

$$\eta = \frac{1}{1 + \frac{1}{2}(mL)^2} \tag{53}$$

Comparing Equations (52) and (53) behaviors, it is possible to show (see Fig.4) that a better efficiency approximation can be obtained by means of the following relation [2]:

$$\eta = \frac{1}{1 + \frac{1}{3}(mL)^2} \tag{54}$$

Analyzing Fig. 4, estimation error using Eq.(54) instead of (52) is less than 10% when mL value is less than 1.5.

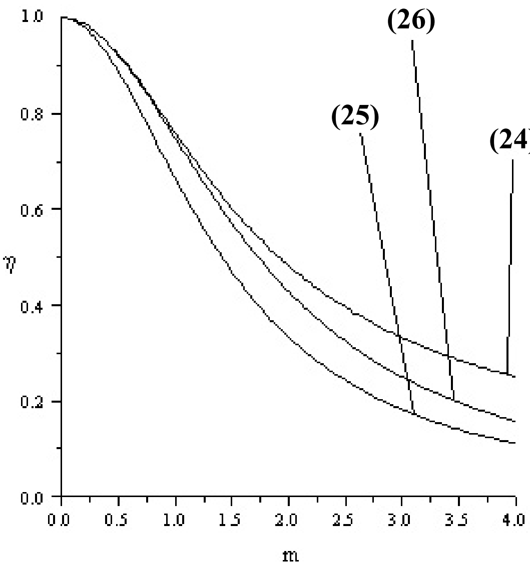

Fig.4: Comparison between behaviors of true fin efficiency and its approximations.

8 Symbols

Symbol	Units	Description
A	m^2	single fin bar final surface area
A_i	m^2	i-th fin wall area
a_i	adimensional	i-th fin wall absorption coefficient
α_i	rad	i-th fin wall emission angle
α_{ij}	rad	i-th fin wall emission angle towards j-th wall
b	m	fin length
β	K^{-1}	thermal dilatation coefficient
C_c	adimensional	fin radiative model calibration parameter
d	m	single fin cavity width
d_A	m	optimal fin spacing obtained by using a unitary fin efficiency
d_B	m	optimal fin spacing obtained with fin efficiency given by Eq. (13)
d_{ott}	m	optimal fin spacing
d_s	m	a fin spacing smaller than the one obtained by the proposed optimization method
ΔT	K	difference between fin and air temperatures
F_{ij}	adimensional	view factor from i-th fin wall to j-th wall
G_i	$W \cdot m^{-2}$	i-th fin wall radiance
g	$m \cdot s^{-2}$	gravity acceleration
γ	adimensional	ratio between fin length and fin spacing
h	$W \cdot m^{-2} \cdot K^{-1}$	fin-air convection coefficient
η	adimensional	fin efficiency
J_i	$W \cdot m^{-2}$	i-th fin wall integral irradiance
$J_{n,i}$	$W \cdot m^{-2}$	i-th fin wall integral irradiance emitted in normal direction
L	m	fin height
λ	$W \cdot m^{-1} \cdot K^{-1}$	fin thermal conductivity
λ_a	$W \cdot m^{-1} \cdot K^{-1}$	air thermal conductivity
m	m^{-1}	fin temperature distribution parameter
n	adimensional	number of fin cavities
Nu	adimensional	Nusselt number
ν	$m^2 \cdot s^{-1}$	cinematic viscosity
P	m	single fin bar perimeter
Pr	adimensional	Prandtl number
Q	W	true exchanged heat flux
Q_d	W	single fin bar heat transfer rate
Q_{id}	W	maximum exchanged heat flux
q	W	heat flux
q_A	W	convective heat flux obtained with known optimization method
q_B	W	convective heat flux obtained with the proposed optimization method
q_c	W	convective heat flux
q_{cav}	W	radiative heat flux emitted by a single fin cavity
q_r	W	radiative heat flux
q_{radi}	$W \cdot m^{-2}$	radiative heat flux relative to wall i
q_s	W	convective heat flux obtained with a fin spacing smaller than the optimal one
R	adimensional	Rayleigh number referred to b
R_{ij}	m	distance between i-th and j-th fin walls
Ra	adimensional	Rayleigh number
Ra'	adimensional	modified Rayleigh number
r_1	adimensional	i-th fin wall reflection coefficient
S	m^2	single fin bar area
s	m^{-1}	fin logarithmic temperature distribution parameter
σ_0	$W \cdot m^{-2} \cdot K^{-4}$	Stefan-Boltzmann constant
T	K	fin temperature distribution
T_0	K	fin walls uniform temperature
T_i	K	fin i-th wall temperature
T_4	K	air wall 4 temperature
T_{ex}	K	air wall temperature

T_L	K	fin upper wall temperature
T_p	K	fin wall temperature
T_s	K	fin cavity surface temperature when x=0
T_∞	K	air temperature
t	m	single fin bar width
u	adimensional	substitution variable
u_{ott}	adimensional	optimum u value
v	m	fin base surface thickness
x	m	portion of fin height
y	adimensional	division between fin cavity width and fin length
z	m	fin width

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